

Influence of initial distributions on robust cooperation in evolutionary Prisoner's Dilemma

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Abstract. - We study the evolutionary Prisoner's Dilemma game on scale-free networks for different initial distributions. We consider three types of initial distributions for cooperators and defectors: initially random distribution with different frequencies of defectors; intentional organization with defectors initially occupying the most connected nodes with different fractions of defectors; intentional assignment for cooperators occupying the most connected nodes with different proportions of defectors at the beginning. It is shown that initial configurations for cooperators and defectors can influence the stationary level of cooperation and the evolution speed of cooperation. Organizations with the vertices with highest connectivity representing individuals cooperators could exhibit the most robust cooperation and drive evolutionary process to converge fastest to the high steady cooperation in the three situations of initial distributions. Otherwise, we determine the critical initial frequencies of defectors above which the extinction of cooperators occurs for the respective initial distributions, and find that the presence of network loops and clusters for cooperators can favor the emergence of cooperation.

Introduction. – Evolutionary game theory has become an important tool for investigating cooperative behavior of biological, ecological, social and economic systems [1, 2]. The Prisoner's Dilemma game (PDG) is one of the most commonly employed games for this purpose. Originally, in the PDG, two individuals adopt one of the two available strategies, cooperate or defect; both receive R under mutual cooperation and P under mutual defection, while a cooperator receives S when confronted to a defector, which in turn receives T , where $T > R > P > S$ and $T + S < 2R$. Under these conditions it is best to defect for rational individuals in a single round of the PDG, regardless of the opponent strategy. However, mutual cooperation would be preferable for both of individuals. Thus, the dilemma is caused by the selfishness of the individuals.

However, the unstable cooperative behavior is opposite to the observations in the real world. This disagreement thus motivates to find under what conditions the cooperation can emerge on the PDG. Graph theory provides a nat-

ural and very convenient framework to study the evolution of cooperation in structured populations. In well-mixed populations, each individual interacts with each other individual. The average payoff of defectors is greater than the average payoff of cooperators and the frequency of cooperators asymptotically vanishes. In other structured populations, each individual occupies one vertex and individuals only interact with their neighbors in a social network. Several studies have reported the cooperation level on different types of networks [3–7]. Nowak and May introduced a spatial evolutionary PDG model in which individuals located on a lattice play with their neighbors, and found that the spatial effect promotes substantially the emergence of cooperation [3]. Santos *et al.* have studied the PDG and Snowdrift game (SG) on scale-free networks and found that comparing with the regular networks, scale-free networks provide a unifying framework for the emergence of cooperation [6]. Notably, scale-free networks where the degree distribution follows a power law form are highly heterogeneous, and the heterogeneity of the network structure can promote cooperation. However,

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the puzzle of cooperation on social networks is unanswered yet. Recently, the roots of the diverse behavior observed on scale-free networks are explored [8,9]. Cooperators can prevail by forming network clusters, where they help each other on heterogeneous networks [10]. In scale-free networks, the majority of nodes have only a few links, while a small number of nodes with high connectivity (hubs) are well connected to each other. This extremely inhomogeneous connectivity distribution results in the robustness of scale-free networks [11]. As a result, the presence of hubs and relative abundance of small loops for cooperators in scale-free networks can promote the level of cooperation.

From these results on scale-free networks, it seems that cooperation can be affected by the initial distribution for cooperators (C) and defectors (D), such as randomly or intentionally distributions, individuals initially assigned with equal or unequal probability to be C or D. Similarly, a special initial distribution for C and D may exhibit a robust cooperation on scale-free networks. However, in most literature, initial strategies of individuals are randomly assigned with the same probability to be C or D. Here, we remove the setting and are interested in investigating the evolution of cooperation for different initial distributions on scale-free networks. The paper is organized as follows. In the next section, we describe the evolutionary game model as well as networks in detail. And then simulation results and analysis are provided in the third section. Finally, conclusions are given in the fourth section.

The model. — Firstly, we construct scale-free networks using the Barabási and Albert model (BA) which is considered to be the typical model of the heterogeneous networks [12]. Starting from m_0 vertices which are connected to each other, at each time step one adds a new vertex with m ($m \leq m_0$) edges that link the new vertex to m different vertices already present in the system. When choosing the vertices to which the new vertex connects, one assumes that the probability P_i that a new vertex will be connected to vertex i depends on the degree k_i of vertex i : $P_i = k_i / \sum_j k_j$. After t time steps this algorithm produces a grape with $N = t + m_0$ vertices and mt edges. Here, we set $m = m_0 = 2$ and network size $N = 3000$ for all the simulations. Thus, the average degree of this network model can be given $\bar{k} = 2m = 4$.

After constructing networks, each site of the network is occupied by an individual. Each individual who is a pure strategist can only follow two simple strategies: cooperate and defect. In one generation, each individual plays a PDG with its neighbors simultaneously, and collects payoffs dependent on the payoff matrix parameters. The total payoff of a certain individual is the sum over all interactions in one generation. Following common practice [3,13], we use a simplified version of PDG, make $T = b$, $R = 1$ and $P = S = 0$, where b represents the advantage of defectors over cooperators, being typically constrained to the interval $1 < b < 2$. Let us represent the individuals' strategies with two-component vector, taking the value $s = (1, 0)^T$

for C-strategist and $s = (0, 1)^T$ for D-strategist. Therefore, the total payoff P_x of a certain individual x can be written as

$$P_x = \sum_{y \in \Omega_x} s_y^T A s_y, \quad (1)$$

where the sum runs over all the neighboring sites of x , and Ω_x is the set of neighbors of element x .

During the evolutionary process, each individual is allowed to learn from one of its neighbors and update its strategy in each generation. Following previous works [13,14], each individual chooses one individual randomly from its neighbors. After choosing a neighbor y , the individual x adopts the selected y neighbor's strategy in the next generation with a probability depending on their total payoff difference as

$$W_{s_x \leftarrow s_y} = \frac{1}{1 + \exp[(P_x - P_y)/K]}, \quad (2)$$

where K characterizes the noise effects, including fluctuations in payoffs, errors in decision, individual trials, etc. And P_x, P_y denote the total payoffs of individuals x and y , respectively. Here, K is set to 0.125 for the total payoffs. Furthermore, the results remain unaffected with different values of the parameter K .

Simulations and discussion. — In the following, we will show the simulation results carried out for a population of $N = 3000$ individuals occupying the vertices of the scale-free networks with $\bar{k} = 4$. The above model is simulated with synchronous updating. Eventually, the system reaches a dynamic equilibrium state. The equilibrium frequencies of C are obtained by averaging over the last 1000 generations after a transient time of 10000 generations. In what follows, three situations of initial distributions for C and D will be considered: (1) defectors are randomly distributed to occupy the network vertices; (2) defectors on purpose occupy the highly connected nodes; (3) defectors are intentionally assigned to occupy the nodes with small connectivity. In these respective situations, the effects of different initial frequencies of defectors f_{ID} on the emergence of cooperation are subsequently investigated, too. In situations (2) [situation (3)], nodes in the scale-free networks are sorted by decreasing (increasing) number of links that each node contains. There are instances where groups of nodes contain identical numbers of links. Where this occurs, they are arbitrarily assigned a position within that groups. For example, the node rank r denotes the position of a node on this ordered list and $1 \leq r \leq N$ [15]. Initially, when r defectors occupy the highly connected nodes, they just occupy the r nodes with highest connectivity in the networks; while r defectors occupy the nodes with small connectivity, they just occupy the r nodes with smallest connectivity in the networks, and thus $f_{ID} = r/N$ is the initial frequency of D. The evolution of the frequency of C as a function of b and f_{ID} for different initial distributions has been computed. To this end, each data point

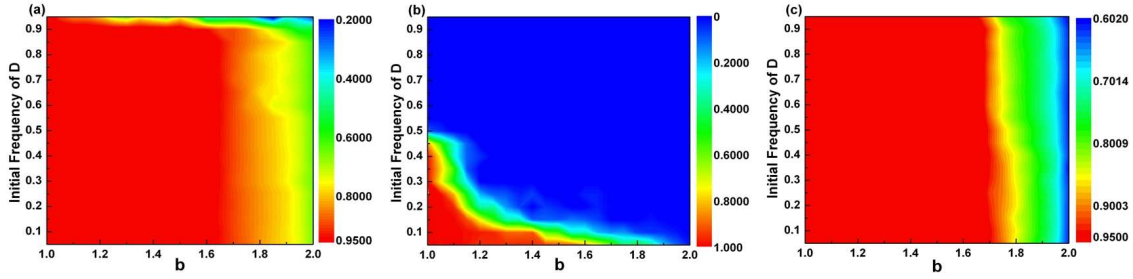


Fig. 1: (Color Online) Evolution of cooperation in scale-free network with $\bar{k} = 4$. Results for the fraction of C at equilibrium in the population are plotted as a contour, drawn as a function of two parameters: b and f_{ID} . (a) random distributions with different initial frequencies of D; (b) different initial fractions of D which occupy nodes with high connectivity; (c) different initial percentages of D which occupy nodes with small connectivity.

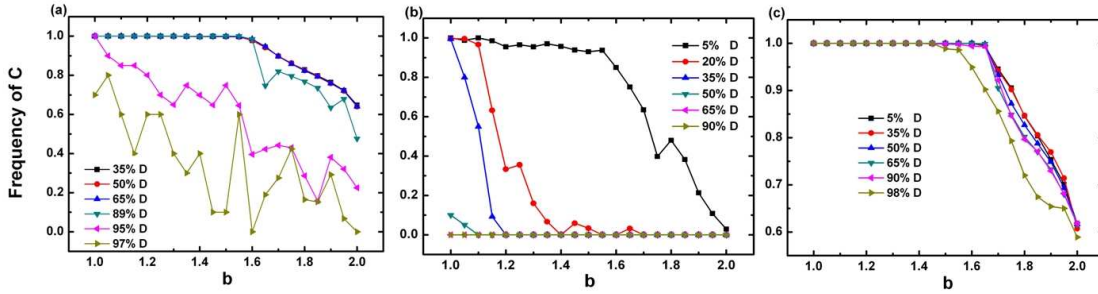


Fig. 2: (Color Online) Frequency of C at equilibrium as a function of the parameter b for different distributions with different values of f_{ID} .

results from an average over 30 realizations of both the networks and same initial distributions.

Fig. 1 shows the simulation results in the PDG for different initial distributions as a contour plot. Clearly, in fig. 1(a) we have found that the cooperation level becomes poorer when the initial frequency of D increases for a given fixed b . Especially, the cooperation level begins to fluctuate and decreases intensively when initial frequency of D is large and near one for high values of b . While cooperators dominate over the most ranges of b and ρ_{ID} in this situation. In fig. 1(b), cooperation strongly depends on the values of f_{ID} , and defectors dominate over the most ranges of b and f_{ID} . In fig. 1(c), a certain amount of cooperation can emerge and remain stable even for high initial frequency of D, and cooperators prevail over the most ranges of b and f_{ID} . A comparison of the different results for different initial frequencies of D is shown in fig. 2. We depict the cooperation level as a function of the parameter b . In fig. 2(a), we have found that the equilibrium frequency of C begins to fluctuate and decrease for high values of b when the initial frequency of D is high. And the cooperation level remains stable for small values of b . Additionally, when f_{ID} increases and approaches one, cooperation fluctuates intensively and cooperators dies out finally. As shown in fig. 2(b), cooperation is strongly inhibited as b increases when defectors are not wiped out. There are larger oscillations and cooperation is sensitive

to initial frequency of D when defectors occupy the nodes with highest connectivity at the beginning. Moreover, cooperators vanish when the initial frequency of D is more than 50% no matter what the value of b is. Fig. 2(c) exhibits a robust and favorable cooperation for different initial frequencies of D. Even if a small number of cooperators initially occupy the rich nodes, it still leads to a high cooperation level. The frequency of C decreases slowly for a high initial frequency of D over the whole region of b . The cooperative behavior is robust against defector's invasion in this situation. From fig. 2, we know that different initial frequencies of D and distributions can result in different levels of cooperation. In addition, in comparison with the two other situations, the situation that cooperators occupy the rich nodes, presents much more robust cooperation in this respect that high cooperation remains for almost any temptation. It is shown that the time evolution of cooperation in PDG for different values of b and initial distributions with the same f_{ID} in fig. 3. It is found that situation (3) that cooperators occupy the most connected nodes at the beginning makes evolutionary process converge much fastest to the equilibrium state of 100% cooperators in the three situations, while situation (2) that defectors firstly occupy the nodes with highest connectivity provides much harsher condition for the emergence of cooperation than the two other situations and makes cooperation level drop much fast. Situation (3) promotes the

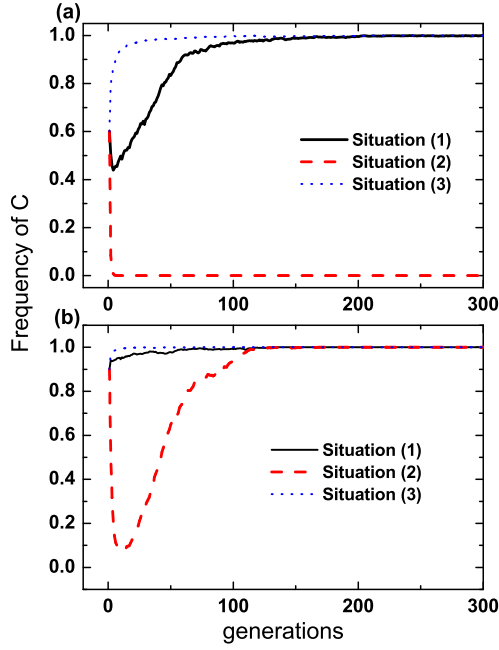


Fig. 3: (Color Online) Frequency of C at equilibrium as a function of evolution generations for different values of b and initial distributions with the same f_{ID} . (a) $b = 1.4$ and 60% cooperators at the beginning; (b) $b = 1.1$ and 90% cooperators at the beginning.

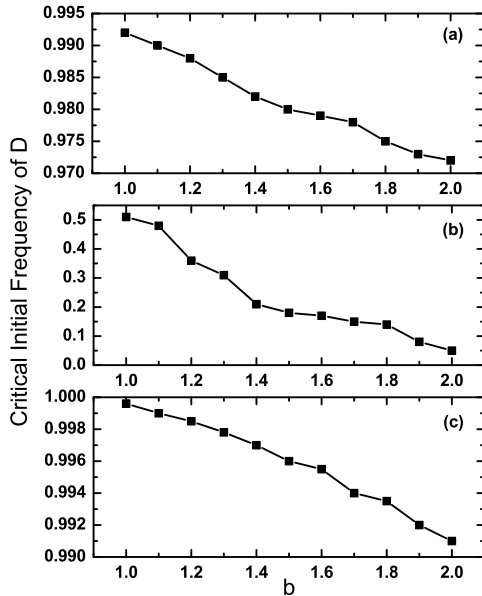


Fig. 4: Critical frequency of D for cooperators to vanish in the PDG as a function of the parameter b for different situations.

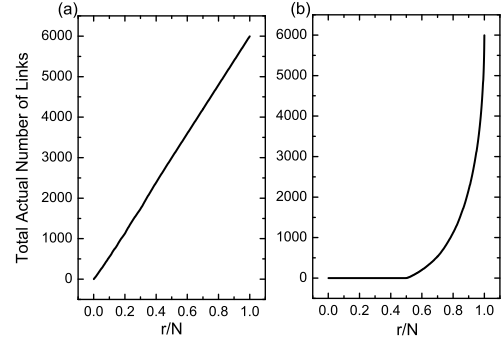


Fig. 5: The total actual number of links among r nodes against r/N with $N = 3000$ and $m = m_0 = 2$ in scale-free networks, where r represents the node rank. (a) r nodes with the highest connectivity in the networks; (b) r nodes with the smallest connectivity in the networks. Each data point of the curves results from 10 different network realizations.

emergence of cooperation and can speed up the evolution of cooperation. Fig. 4 shows the critical frequency of D for cooperators to vanish in the PDG as a function of b for the three types of initial distributions and also illuminates these results. When the initial frequency of D is higher than the critical frequency of D, cooperators vanishes or decreases intensively to extinction. For an arbitrary value of b , the critical frequency of D in situation (3) is always higher than those in situation (1) and (2). Initial ratios of C in one certain distribution for C and D can affect the cooperation level; otherwise, initial distributions for C and D also influence the emergence of cooperation and the evolution speed of cooperation.

These simulation results can be understood in the following way. In scale-free networks, there are a large number of nodes which have only a few links, and there are small number of links among these less connected nodes; while there are a small number of nodes with large numbers of links, these most connected nodes or hubs are generally very well connected to each other (see fig. 5). The connectivity between these hubs in the networks can be crucial for the emergence of cooperation for the PDG [9–11, 14, 16]. Based on these results, some corresponding explanations on our results can be provided. At first, we discuss the random initial distribution with different fractions of D. When the initial percentage of D is small, nodes with high connectivity will be occupied by defectors with much smaller probability. In this case, individuals using strategy C representing highly connected nodes communicate with each other and form loop and main cluster structures, and hence the high levels of cooperation can emerge. Therefore, the probability, with which most connected vertices are occupied by cooperators, decreases when the initial fraction of D increases. Then clusters of cooperators may be cut off (fragmented) from the main compact cluster, but there are still some loops and frag-

ments for cooperators. In this state there is a systematic drop of cooperation at the beginning, nevertheless it tends to rise again in the long run, thereby, cooperation falls but can remain at a high level. While the initial percentage of D is more than the critical frequency, it is still possible for a small number of C players to occupy the nodes with high connectivity although the probability is so small, since strategies C and D are randomly distributed among all the players. Thus, there are large oscillations when the initial frequency of D approaches one, because it is increasingly difficult for the cooperators occupying most connected nodes to communicate with each other in this state. And then we investigate the situation that defectors occupy the nodes with highest connectivity at the beginning. In other words, cooperators initially occupy vertices having only a few links. In fig. 5(b), it shows that there are few actual links among about half of the nodes which are almost the least connected nodes. When the initial frequency of D is more than 50% in situation (2), it is not possible to form network clusters for cooperators where cooperators can help each other, and defectors are grouped in several clusters, then cooperators lose more and more elements from their outer layer along with the increment of evolution generations, therefore, cooperators can not survive no matter what the value of b is. Nevertheless, only small isolated pieces can be formed for cooperators when the fraction of D is less than the critical frequency, since defectors occupy the most connected nodes. Thus, it results in that cooperation falls intensively and can not remain stable. However, a high level of cooperation is sustainable just for small values of b , because in this case defectors have not much advantage over cooperators. For $b \sim 1$, cooperators are equivalent to defectors, then the level of cooperation is not strongly susceptible to the initial distribution for C and D. In fact, in all generations cooperation falls rapidly at the beginning, then cooperators sometimes recover but not always for small values of b in situation (2). For large b , cooperation always falls and never recovers. Therefore, in this state cooperation drops rapidly and it needs much time to revert cooperation if cooperation can recover finally. Accordingly, cooperators vanish at the stationary state over the most regions of b and f_{ID} . The situation, that individuals C intentionally assigned to represent the vertices with high connectivity at the beginning, is analyzed finally. In this case, it is easy for cooperators to form giant compact network clusters and loops. Even if a small number of cooperators occupy the most connected nodes, there are a large number of loops and some tiny compact clusters for cooperators; conversely, defectors are not organized in these clusters, where cooperators can help each other and defectors can not invade. The presence of clusters and loops in the connectivity structure for cooperators sustains the high level of cooperation even for a high value of b , and in all generations they can favor cooperation at the beginning, and drive evolutionary process to converge fast to the high stationary cooperation level. Therefore, coopera-

tors dominate over the entire ranges of b and f_{ID} , and the cooperative behavior is robust against defectors' invasion in this situation.

Scale-free networks have most of their connectivity clustered and looped in a few nodes, therefore, initial assignments for C and D can affect the cooperative behavior and the evolution speed of cooperation. The configuration that cooperators initially occupy the most connected nodes, presents the much more robust cooperation than the two other ones and can speed up the evolution of cooperation, in comparison with the two other different initial configurations. Moreover, cooperators can prevail by forming network clusters and loops, where they can assist each other. These results are independent of the size of the populations N .

Conclusions. — In summary, we have studied the cooperative behavior of the evolutionary PDG on scale-free networks for different initial distributions, and also found that the presence of network loops and clusters for cooperators can favor cooperation. Cooperators dominate over the most range of b with different initial frequencies of D when strategies C and D are randomly distributed among the populations; a poor and unstable cooperation level can be established at equilibrium in the state that the vertices with high connectivity represent defectors at the beginning; while a very robust and favorable cooperation can be exhibited in the situation that the highly connected nodes are occupied by cooperators at the beginning. The situation that cooperators initially occupy the most connected nodes provides the most robust cooperation in the three situations of initial distributions for C and D. Additionally, it is found that the configuration that cooperators occupy the most connected nodes at the beginning can speed up the evolution of cooperation; while the situation that defectors occupy the most connected nodes drives cooperation to drop fast and difficultly recover. And the critical frequencies of D for cooperators to vanish corresponding to initial distributions have been respectively determined. Some qualified explanations based on the property of scale-free networks are given for these phenomenon. Therefore, our results shows that initial configurations for C and D with different ratios of D at the beginning can affect the levels of cooperation on heterogeneous networks. Moreover, our work may be helpful in exploring the roots of the emergence of cooperation on heterogeneous networks.

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